**Microscopic-macroscopic nuclear level densities for low excitation energies**

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The statistical level density is a fundamental tool for the description of many properties of atomic nuclei. The level density $\rho(E,N,Z,M)$ of a nucleus is defined as function of the energy $E$, neutron $N$ and proton $Z$ numbers, and the projection of the angular momentum $M$ on a laboratory-fixed coordinate system. The level density $\rho(E,N,Z,M)$ can be presented as the inverse Laplace transformation of the partition function $Z(\beta,\alpha)$, where $\beta$ and $\alpha=\{\alpha_N,\alpha_Z,\alpha_M\}$ are the Lagrange multipliers. These multipliers are determined by the neutron $N$ and proton $Z$ numbers, and the spin projection $M$, respectively. Within the grand canonical ensemble, one can apply the standard Darwin-Fowler saddle point method (SPM) for the inverse Laplace integration over all variables, including $\beta$, which is related to the total energy $E$, but for large excitation energy $U$. As it can be assumed, the temperature $T$ is related to the well-determined saddle point in the integration over $\beta$ for a finite Fermi system of large particle numbers $N$ and $Z$, and the spin projection $M$. However, many experimental data also exist for low-lying excitation energies $U$ where such a saddle point does not exist. Therefore, the integral over the Lagrange multiplier $\beta$ should be carried out [1] more accurately beyond the standard SPM. For other variables, related to the neutron $N$ and proton $Z$ numbers, and the spin projection $M$, one can apply the SPM assuming that particle numbers $N$ and $Z$, and spin $I$ are relatively large. In this work we have removed divergence at the critical catastrophe point at zero excitation energy limit, where all high-order derivatives of the entropy are zeros, using the semiclassical periodic-orbit theory (POT) for calculations of $\rho(E,N,Z,M)$, and have derived analytical expressions for the spin-dependent level density $\rho(E,N,Z,I)$ for the nuclear collective rotations.

Taking the inverse Laplace transformation of the partition function $Z(\beta,\alpha)$ over $\beta$ more accurately, beyond the standard SPM, we have derived [1] an approximate expression for the level density, $\rho(E,N,Z,M)\propto S^{-\nu}I_\nu(S)$, where $I_\nu(S)$ is the modified Bessel function of the entropy $S$ at $\nu=5/2$ for the MMA1 case (i) of negligibly small shell corrections, and $\nu=7/2$ for the MMA2 case (ii) of the dominating shell effects. For the spin-dependent level density $\rho(E,N,Z,I)$, one obtains $\rho(E,N,Z,I)\propto a\hbar^2(2I+1)S^{-(\nu+1)}I_{\nu+1}(S)/\Theta$. The shell and isotopic asymmetry effects are taken into account through the level density parameter $a$ and nuclear moment of inertia $\Theta$. For collective rotations one obtains $\rho_{\rm coll}(E,I)\approx(1/2)\int_{-I}^{I}d\,\Lambda\,\rho_{\rm intr}(U_0-E^{\perp}_{\rm rot},\Lambda)$, where $\rho_{\rm intr}(U_{\rm coll},\Lambda)\equiv\rho(E,M=\Lambda)$ is the intrinsic level density, $\Lambda$ is the nuclear spin projection to the intrinsic axis of the coordinate system rotating together with a nucleus, $U_0=E-E_0$, $E_0$ is the background energy, $U_{\rm coll}=U_0-E^{\perp}_{\rm rot}$, $E^{\perp}_{\rm rot}=[I(I+1)-\Lambda^2]/2\Theta_\perp$ is the rotation energy and $\Theta_\perp$ is the moment of inertia with respect to the axis perpendicular to the symmetry axis. As the "parallel" moment of inertia $\Theta_\parallel$ is much smaller than $\Theta_\perp$ of the total effective moment of inertia $\Theta_{\rm eff}$, $\Theta^{-1}_{\rm eff}=\Theta^{-1}_\perp+\Theta^{-1}_\parallel$, we have proved analytically the well-known enhancement of the level density due to the collective rotations of axially deformed symmetric nuclei. For the total MMA level density $\rho(E,N,Z)=\int dM\,\rho(E,N,Z,M)$ we arrived at a similar expression, $\rho(E,N,Z)\propto S^{-\nu}I_\nu(S)$, but with $\nu=2$ for the MMA1 case (i) and $\nu=3$ for the MMA2 case (ii).

Fig. 1 shows the inverse level density parameter $K = A/a$ in the long Nd isotope chain with the particle number $A = 131 - 156$. The values of $K$ are obtained by the least mean squares (LMS) fitting of our theoretical results to the experimental data on the known excitation spectra found by the sample method from the database, http://www.nndc.bnl.gov/ensdf. A reasonable agreement with these experimental data are obtained accounting for the shell and isotopic asymmetry effects with the help of the semiclassical POT by using the only one inverse level density parameter $K$, having a clear physical meaning, in the LMS fitting. As seen from Fig. 1, the results for $K$ for isotopes of Nd as functions of the particle number $A$ are characterized by a very pronounced saw-toothed behavior with alternating low (for odd nucleus) and high $K$ values (for even nucleus). We obtained values of $K$ for low excitation energy range which are essentially different from those for neutron resonances $K \approx 10$ MeV. We have found significant shell effects in the MMA level densities mainly because of dominating contributions from MMA2b (ii) approach, which is the MMA2 (ii) but for small shell corrections and for their large derivatives over the chemical potential, in the nuclear low-energy states range. The interparticle interaction beyond the mean field approach is taken into account approximately in average through the extended Thomas-Fermi component of the level density parameter $a$, proportional to the semiclassical POT level density. Accounting for the strong pairing effect (two magenta points for $^{140,142}$Nd in Fig. 1) we showed that pairing correlations lead to a smoothing of the inverse level density $K(A)$ behavior as function of the particle number $A$.

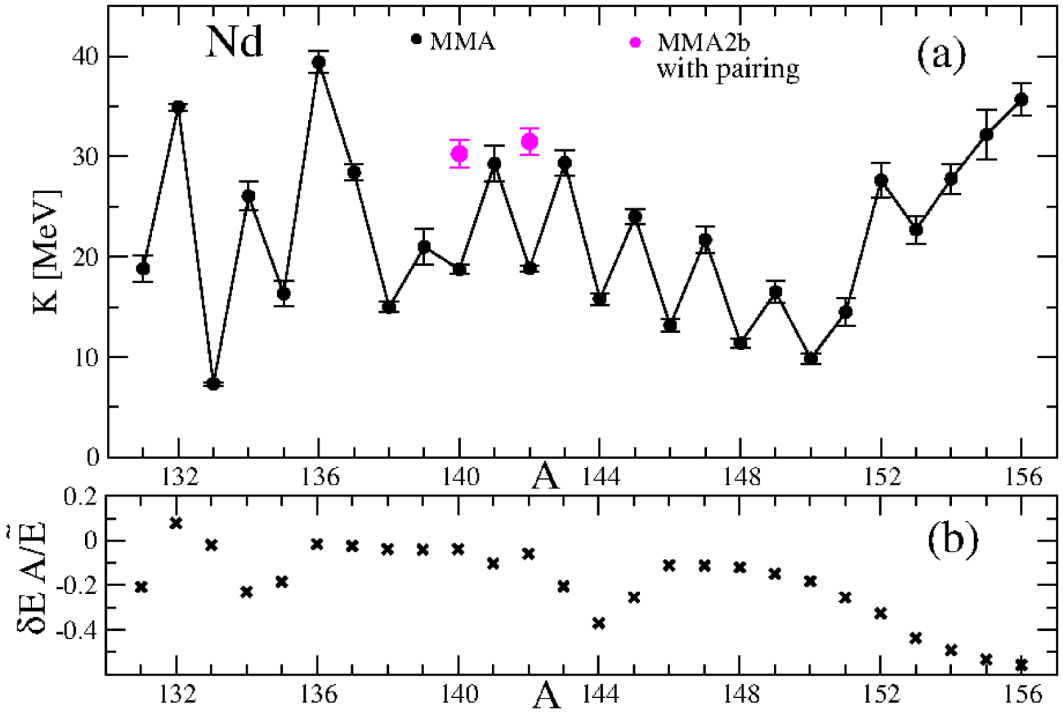


**Fig. 1.** (a) Inverse level-density parameter $\boldsymbol{K}$(with errors bars) for Nd isotopes is shown as function of the particle number $\boldsymbol{A}$ within a long chain $\boldsymbol{A = 131 - 156}$. The close black dotted points are the results of the MMA approach taken with the smallest relative error parameter, $\boldsymbol{\sigma^2 \propto \chi^2}$ , normalized to the number of points, in the LMS fitting among all MMAs (MMA1, MMA2a, and MMA2b), including MMA2a which is MMA2(ii) but with the numerical shell corrections taken from Ref. [2]. (b) The relative shell correction energies , $\boldsymbol{\delta E\, A/\tilde{E}}$ [2], in units of the background energy $\boldsymbol{\tilde{E} \approx E_{ETF}}$ per one particle, $\boldsymbol{E_{ETF}}$ is the smooth energy POT component of the extended Thomas-Fermi model. The chemical potential is $\boldsymbol{\lambda = 40}$ MeV.

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